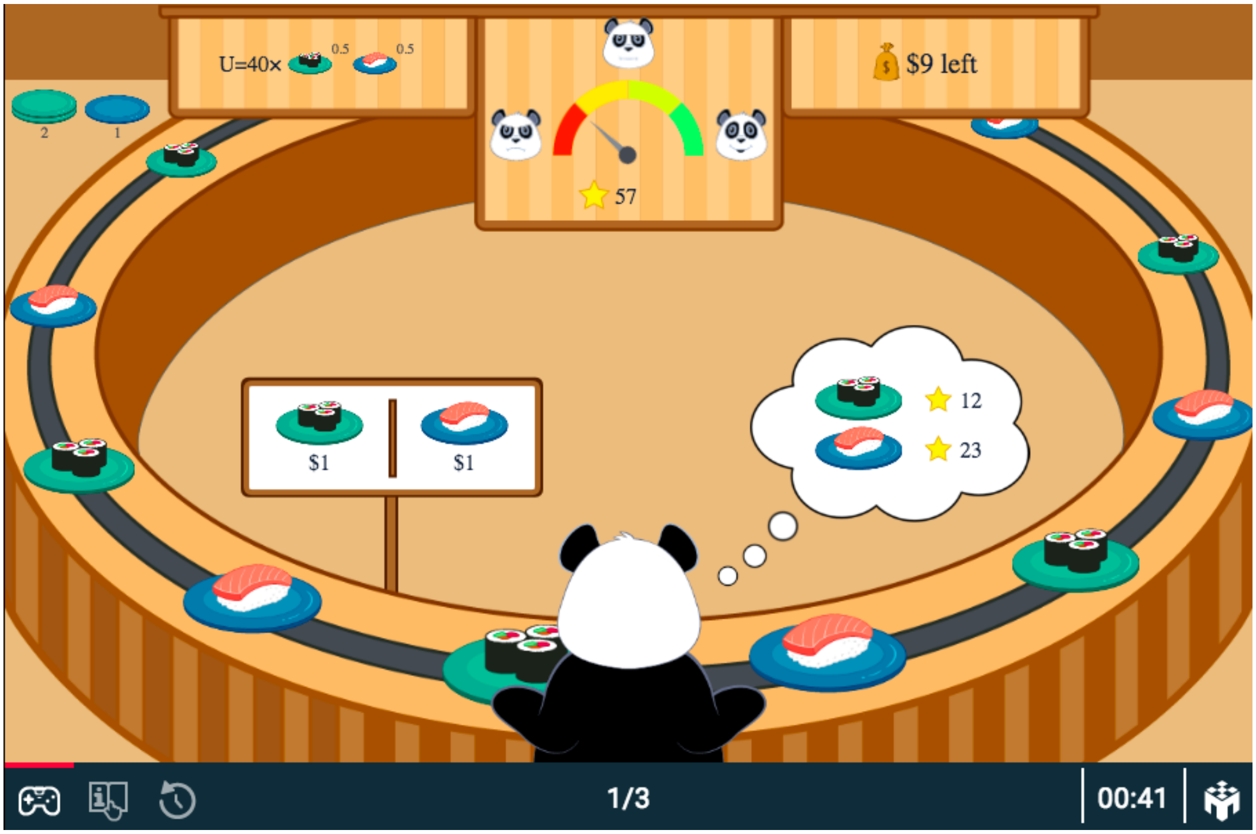
Utility Maximization



1. Start by labeling the important components of the utility maximization problem in the picture above. What is the Panda's goal? What resources are available to achieve that goal? What costs incurred?

2. If $M = 12$ and the price of x (p_x) is \$1 and the price of y (p_y) is \$1.

a. Draw the budget constraint this consumer faces

b. Most people obtain the optimal bundle (x,y) = (6,6) in Rounds 1 - 3. These yields a total utility of $U(x, y) = 40x^{1/2}y^{1/2} = 40 * 6^{1/2} * 6^{1/2} = 240$. Overlay three indifference curves on the same graph such that one indifference curve is sub-optimal, one indifference curve contains the optimal bundle, and another indifference curve contains bundles that would be preferred but cannot be afforded.

c. Using graphical analysis, explain the intuition for why (6,6) was the optimal bundle.

3. We want to think in terms of utility. To make optimal decisions we must think on the margin. What is marginal utility?

4. One important transitional idea is that we can think about our MRS of $\frac{\Delta Y}{\Delta X}$ as $\frac{MUx}{MUy}$. Why is that?

5. Now we want to build understanding about the key elements of the utility maximization problem. The key idea is that $MRS_{x,y} = \frac{p_x}{p_y}$. But, that likely doesn't make sense quite yet.

a. Given a utility function $U(x, y) = 40x^{1/2}y^{1/2}$, what is the marginal utility of x (take the derivative)?

b. Given a utility function $U(x, y) = 40x^{1/2}y^{1/2}$, what is the marginal utility of y (take the derivative)?

c. What is the $MRS_{x,y}$ for this utility function?

d. What is the price ratio (i.e. the opportunity cost)?

6. Divide the marginal utility for good X by the price of good X. Divide the marginal utility for good Y by the price of good Y.

(a) When $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$, what does that imply?

(b) When $\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$, what does that imply?

(c) When $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$, what does that imply?

7. Now that we have cemented the intuition for this problem, let us verify using utility maximization that indeed (x,y)=(6,6) is the optimal bundle.

8. If $M = 12$ and the price of x (p_x) is \$2 and the price of y (p_y) is \$1.

a. Draw the budget constraint this consumer faces

b. Most people do not obtain the optimal bundle (x,y) = (3,6) in Rounds 4 - 6 because they "chase" marginal utilities. But, in part 5 of this worksheet we saw why this is only part of what needs to be considered. For those that chased marginal utilities and didn't pay attention to prices that yielded a total utility $U(x, y) = 40x^{1/2}y^{1/2} = 40* 4^{1/2} * 4^{1/2} = 160$ rather than the 170 total utility that the bundle (3,6) would have attained.

c. Overlay three indifference curves on the same graph such that one indifference curve is sub-optimal, one indifference curve contains the optimal bundle, and another indifference curve contains bundles that would be preferred but cannot be afforded.

9. Using these new prices, let us verify using utility maximization that indeed $(x,y)=(3,6)$ is the optimal bundle.

10. One property of Cobb-Douglas utility is that (in equilibrium) the exponent share $\frac{a}{a+b}$ equals the budget share $\frac{xp_x}{M}$. In your own time, show that this is true with a utility function $U(x, y) = x^a y^b$ and a budget constraint $p_x x + p_y y = M$. What is the economic logic of the result (i.e. why does it make sense)?

11. All else equal, how would the optimal bundle change if utility were described by the function $U(x, y) = Ax^a y^b$, where A=100?

12. All else equal, how would the optimal bundle change if utility were described by the log utility of $U(x, y) = x^a y^b$, which would be $U(x, y) = a * log(x) + b * log(y)$?

13. What was the most important thing you learned today? What questions still remain in your mind?